NEW DOCTORAL DEGREES

IN THE DEPARTMENT OF MATHEMATICS UNIVERSITY OF OSIJEK

Dr. Krešimir Burazin received his PhD in Mathematics from the Department of Mathematics of the University of Zagreb on 30 September 2008 with the dissertation entitled "Contribution to the theory of Friedrichs' and hyperbolic systems" (Mentor: Dr. N. Antonić).

Abstract

We study various topics concerning Friedrichs' systems (first two chapters) and first-order semilinear hyperbolic systems (third chapter).

In the first part the basic properties of graph spaces of first-order differential operators are given, with special emphasis on investigation of properties of the trace operator, which plays a major role when imposing boundary conditions. Here we follow Ph.D. thesis of M. Jensen (2004), with proofs being simplified and shortened.

In the second part we study Friedrichs' systems, a class of boundary value problems that admit the study of a wide range of differential equations in a unified framework. They were introduced by K. O. Friedrichs in 1958 as an attempt to treat equations of mixed type (such as Tricomi equation). In a recent paper: A. Ern, J.-L. Guermond, G. Caplain: *An Intrinsic Criterion for the Bijectivity of Hilbert Operators Related to Friedrichs' Systems*, Comunications in Partial Differential Equations **32**(2007), 317–341; a new view on the theory of Friedrichs systems was given, as the theory is written in terms of Hilbert spaces, and a new way of representation of boundary conditions was introduced. Here, the admissible boundary conditions are characterized by two intrinsic geometric conditions in graph space, which avoids invoking traces at the boundary. Authors show that these conditions imply maximality of boundary conditions. They also introduce another representation of boundary conditions via a boundary operator, and show that this representation is equivalent with intrinsic one (that enforced by two geometric conditions) if two specific operators P and Q exist. We have noted that these two geometric conditions can be naturally written in terminology of an indefinite inner product on graph space, and use of classical results in Krein spaces allowed us to construct the counter–example, which shows that operators P and Q do not always exist. We also investigate situations when the existence of P and Q is guarantied – the case of one space dimension. By using Kreine space we show that maximality of the boundary condition implies intrinsic (geometric) conditions, and give a more elegant proof for the converse statement.

The relation between a *classical* representation of admissible boundary conditions (via matrix fields on boundary), and those given by the boundary operator is addressed as well: necessary conditions on the boundary matrix in order to define the boundary operator with satisfactory properties are given, followed by some examples.

The third part is concerned with first-order decoupled semilinear hyperbolic systems. The local existence and uniqueness result can be found in L. Tartar: *From Hyperbolic Systems to Kinetic Theory, A Personalized Quest*, Springer-Verlag, Berlin Heidelberg 2008, and it is paired with an estimate on the solution of a certain type. We prove it under slightly generalized assumptions that do not change the proof, but allow a more precise estimate on the solution. The estimates on the solution and the time of its existence is the main topic of this chapter. It is shown how to achieve the best possible estimate on the solution and its time of existence (the best among all estimates of a certain type—the type provided by the existence and uniqueness theorem). The L^p version (for $1 < p < \infty$) of the existence and uniqueness theorem is also briefly discussed.

Published papers

[1] N. Antonić, **K. Burazin**, *Graph representation for asymptotic expansion in homogenisation of nonlinear first-order equations*, Annali dell'Universita di Ferrara. Sezione VII. Scienze Matematiche. **53**(2007), 149–176.

[2] **K. Burazin**, *Estimates on the weak solution of semilinear hyperbolic systems*, Annali dell'Universita di Ferrara. Sezione VII. Scienze Matematiche. **54**(2008), 229–243.

[3] N. Antonić, **K. Burazin**, *Graph spaces of first-order linear partial differential operators*, Math. Commun., 2009, accepted for publication.

[4] N. Antonić, **K. Burazin**, *On certain properties of spaces of locally Sobolev functions*, Proceedings of the conference on Appl. Mathematics and Scientific Computing, (Z. Drmač, M. Marušić and Z. Tutek, Eds.) Springer, 2005, 109-120.